

Fuzzy linear programming for integrated farming systems in multi-objective environment

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ABSTRACT

Crop production alone from predominantly small farms in India is quite inadequate to sustain the farm families. They have to depend on other land based enterprises, viz. crop, livestock, fishery, poultry, duckery, agroforestry etc. to meet the multifarious needs of farm family. Integrated Farming Systems (IFS) provides scope to integrate different enterprises with the objectives to generate additional farm income and employment, and improve the socio-economic condition of small and marginal farmers. IFS operate under different physical, biological, socio-economic and technological environments. Designing a suitable IFS becomes complex due to the presence of uncertainties such as productivity, market prices of farm produce, non-availability of capital and labour at appropriate time. In the present study, Fuzzy Linear Programming (FLP) is used for developing suitable compromise integrated farming system models for farmers in north Indian situations under multi-objective environment. Based on the socio-economic survey, three objectives, capital requirement (CR), labour employment (LE) and farm income (FI) are considered for development of model. All three objective functions are represented by linear membership functions in fuzzy multi-objective framework. It is observed from compromise solution obtained by FLP that capital requirement, labour employment, farm income are ₹493 071, 897 man-days, ₹604 860, respectively with degree of satisfaction (λ) 0.462. Analysis of compromise solution in multi-objective environment also indicated that capital requirement, labour employment and farm income have differed considerably as compared to individual optimal solutions obtained by solving linear programming individually for each objective function. Sensitivity analysis studies indicated that effect of linear/nonlinear membership functions is having significant effect on degree of satisfaction.

Key words: Fuzzy linear programming, Integrated farming system, Multi-objective environment

Indian agriculture is dominated by small and marginal farmers constituting more than 80% of farming community (ARDB, 2010). These farmers are the core of Indian economy. Crop production alone from small farms is quite inadequate to sustain these farm families. They have to depend on the land based Integrated Farming Systems (IFS) involving crop, livestock, fishery, poultry, duckery, agroforestry etc. to meet the multifarious needs of farm families, like food, fodder, cash, employment etc (Behera *et al.*, 2008). Integrated Farming Systems operate under different physical, biological, socio-economic and technological environments. In addition, combination of different enterprises in integrated farming system requires more scientific and systematic approach for optimal use of resources in the farming systems. Planning of integrated

farming systems becomes more complex due to the presence of uncertainties in the form of productivity, market prices of farm produce and non-availability of capital and labour at appropriate time. Fuzzy programming methodology can be used to handle such situations partially or fully (depending on the given situation) in objective function(s) or constraints or both. These aspects were discussed in detail by Sasikumar and Mujumdar (1998) for water resources related problems. However, application of fuzzy programming to integrated farming system is limited. Fuzzy programming, specifically fuzzy linear or nonlinear programming is having advantages over the other existing multi-objective optimization methods in terms of requirement of incorporating only one additional constraint (with addition of one additional objective function) and less sensitivity analysis requirement. In the present study, Fuzzy Linear Programming (FLP) is explored for developing suitable compromise solution in integrated farming system models in multi-objective environments with a set of farm

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constraints under north Indian situations.

MATERIALS AND METHODS

The study analyses the farming systems of north Indian situation and attempts mathematical modeling to IFS in single objective(s) framework and Fuzzy Linear Programming methodology in multi-objective framework. For this study, socio-economic surveys were conducted in 7 villages (Hyatpur and Kotli villages in Ludhiana district in Punjab, Bhurgrahi village in Goutam Budha Nagar district in Uttar Pradesh and Sonali village in Panipat district and Chahalka, Bhango and Biwan villages in Nuh district in Haryana) in north India during 2005-06 to 2007-08 using both Rapid Rural Appraisal and Participatory Rural Appraisal techniques (FAO 1995, Loader and Amartya 1999). Resource availability and interactions of the different components within the farming system were recorded for each farm by questionnaire. Cost of production and output for each enterprise, and resource flow between the enterprises and interaction of various enterprises were also recorded. The farming systems in the area are characterized as mostly rice-wheat cropping system. Following pre-testing, a structured questionnaire was applied to a total of 100 farms. The data were used for modelling IFS using Linear Programming and FLP during 2008 at the Department of Civil Engineering, Birla Institute of Technology and Science, Pilani, Rajasthan. Proposed models were developed using LINGO (www.lingo.com) optimization software. The data available in Gill *et al.* (2005) is also used for formulation of objective function and constraints.

Different weight scenarios for decision variables i.e. more priority to capital requirement minimization and labour requirement minimization and farm income maximization and equal priority to all criteria, and the bounds for the model are decided based on the farmers situation and need emerged from the socio-economic survey.

A general problem of north Indian situations (Gill *et al.*, 2005), where rice-wheat has become a dominant cropping system for quite a long period and this continuous cropping period has led to severe degradation of land and environment, was taken in to consideration for developing the model. There has been fatigue in production and decline in farm income irrespective of farm sizes (Sidhu *et al.*, 2007). Resource characterization and farm constraints are identified based on the socio-economic survey and included in the model. The survey further revealed that there has been increase in production cost due to escalation of input cost. The rice-wheat has become main source of income for the majority of the farmers. There are four categories of farmers existing in the region. They are marginal, small, medium and large depending on the size of

the holding; on an average they possesses a land holding of 1.0, 2.0, 4.0 and 10.0 (or more) ha, respectively. Marginal and small farmers are dominant with more than 60% of the total farming community (ARDB, 2010). Migration is a common feature for the small and marginal farmers due to non-availability of farm employment and generating employment for them at farm level is a major challenge. They have the surplus labour, while in case of medium and large farmers; labour availability for farm operations is a major problem. They largely depend upon hired labour for their farm activities. On an average marginal, small, medium and large farmers have capital availability for different farm activities round the year of ₹75 000, ₹150 000, ₹300 000 and ₹600 000, respectively. In the present study large farm scenario is analyzed with the following objectives : (i) maximization of farm income, (ii) minimization of hired labour employment and (iii) minimization of capital requirements. These objectives are incorporated in the model for developing IFS strategies in multi-objective framework.

Modeling in single objective (s) framework

Three objective functions i.e., capital requirement (CR), labour employment (LE) and farm income (FI) are optimized individually using Linear Programming (LP) algorithm (Loucks *et al.* 1981). Resulting solution provides allocation of land to various enterprises and corresponding objective function values and provides the basis for lower and upper limits that can be achieved for each objective. These are essential inputs for multi-objective framework.

The model is subjected to constraints related to resource availability such as land (ha), capital (₹), labour (man-days); enterprise options such as Dairy (1 unit equivalent to 0.08 ha), Piggery (1 unit equivalent to 0.0025 ha), Poultry (1 unit equivalent to 0.05 ha), fishery (ha), agro forestry (ha), crop (ha). These six enterprises are represented as X_1 to X_6 respectively for modeling purpose.

Mathematical expression for Linear Programming can be described as (Taha, 2005).

$$\text{Maximise } Z = C X \quad (1)$$

$$\text{Subject to } A X \leq B \quad (2)$$

$$X \geq 0 \quad (3)$$

where $A=(m \times n)$ matrix of known constants; $B=(m \times 1)$ vector of resources; $C=(n \times 1)$ vector of known constants (coefficient of relevant decision variables in the objective function Z); $X=(n \times 1)$ vector of decision variables. Mathematical model for integrated farm system in LP framework is as follows:

Objective function 1: minimization of capital requirements

$$\text{Min : CR}(X) = 178220.5 x_1 + 4624550 x_2 + 3184555 x_3 + 39137.7 x_4 + 19593.56 x_5 + 19879.25 x_6 \quad (4)$$

Objective function 2: minimization of labour employment

$$\text{Min: LE}(X) = 456 x_1 + 13680 x_2 + 1216 x_3 + 34.2 x_4 + 52.8 x_5 + 105 x_6 \quad (5)$$

Objective function 3 : maximization of farm income

$$\text{Max: FI}(X) = 96405 x_1 + 14760900 x_2 + 514113 x_3 + 59132 x_4 + 18406 x_5 + 53221 x_6 \quad (6)$$

Constraints

Restrictions on land

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \leq 10; \quad (7)$$

Restrictions on capital availability

$$178220.5 x_1 + 4624550 x_2 + 3184555 x_3 + 39137.7 x_4 + 19593.56 x_5 + 19879.25 x_6 \leq 600000; \quad (8)$$

Restrictions on labour availability

$$456 x_1 + 13680 x_2 + 1216 x_3 + 34.2 x_4 + 52.8 x_5 + 105 x_6 \leq 1200; \quad (9)$$

Restrictions on farm income

$$178220.5 x_1 + 4624550 x_2 + 3184555 x_3 + 39137.7 x_4 + 19593.56 x_5 + 19879.25 x_6 \leq 532210 \quad (10)$$

Bounds

Dairy: $x_1 = 0.4$;

Piggery: $x_2 = 0.01$;

Poultry: $x_3 = 0.05$;

Fishery: $1.0 \leq x_4 \leq 4.0$;

Poplar (agro forestry) : $1.0 \leq x_5 \leq 4.0$;

Rice-wheat: $2.0 \leq x_6 \leq 4.0$; (11)

Fuzzy Linear Programming Methodology in Multi-objective Framework

Fuzzy Linear Programming (FLP) problem represents data in the form of membership functions with the basis that objectives/constraints in uncertain situation can be modelled by fuzzy sets (Zimmermann, 1996; Ganesh, 2007). Membership functions are based on the chosen shape and tolerance range (varies from maximum and minimum values of the objectives/resources of the constraints). In the present study, only objective functions are considered as fuzzy and membership functions chosen are linear. However, nonlinear membership functions are also studied as part of sensitivity analysis. In the fuzzy linear programming, the problem can be restated as

Find X such that

$$C X \leq Z \quad (12)$$

$$A X \leq B \quad (13)$$

$$\text{and } X \geq 0 \quad (14)$$

Here $\leq$ denotes fuzzified version of $\leq$ and has the linguistic interpretation $\leq$ “essentially smaller than or equal”.

The membership function of the fuzzy set decision model $\mu_D(X)$ is defined as

$$\mu_D(X) = \min_z \{\mu_z(X); z = 1, 2, \dots, O_f\} \quad (15)$$

$\mu_z(X)$ can be interpreted as the degree to which X fulfils the fuzzy inequality $C X \leq Z$. O_f is the number of objective functions. The membership function $\mu_z(X)$ is represented as

For maximization (such as farm income)	For minimization (such as capital requirement)
$\mu_z(X) = 0 \text{ for } Z \leq Z_L$	$\mu_z(X) = 1 \text{ for } Z \leq Z_L$
$\mu_z(X) = \left[\frac{Z - Z_L}{Z_U - Z_L} \right]^\beta \text{ for } Z_L \leq Z \leq Z_U$	$\mu_z(X) = \left[\frac{Z_U - Z}{Z_U - Z_L} \right]^\beta \text{ for } Z_L \leq Z \leq Z_U$
$\mu_z(X) = 1 \text{ for } Z \geq Z_U$	$\mu_z(X) = 0 \text{ for } Z \geq Z_U$

where, Z_U, Z_L are highest and lowest acceptable levels of the objective (represented as * and ** in Table 1) that can be obtained with individual optimization. $\mu_z(X)$ reflects the degree of achievement. Value of $\mu_z(X)$ will be 1 for perfect achievement and 0 for no-achievement of a given strategy and some intermediate values otherwise as described in equation 16. β = Exponent indicating the desired shape of membership functions i.e., linear/nonlinear. Assigning value of 1 to β gives rise to linear membership function whereas any other value gives rise to nonlinear membership function (Sasikumar and Mujumdar, 1998). In the planning scenario, decision maker is interested in a crisp optimum solution rather fuzzy set.

Maximising, Equation (15) becomes

$$\text{Max}_{x \geq 0} \mu_D(X) = \text{Max}_{x \geq 0} \min_z \{\mu_z(X)\}; z = 1, 2, \dots, O_f \quad (17)$$

The model can be restated as (equations 18-20)

$$\text{Max}_{x \geq 0} \min_z \mu_z(X); z = 1, 2, \dots, O_f \quad (18)$$

Subject to

$$A X \leq B \quad (19)$$

$$X \geq 0 \quad (20)$$

Introducing a new variable λ representing degree of satisfaction, Fuzzy Linear Programming problem in multi-objective framework can be formulated as (equations 21-25):

$$\text{Max } \lambda \quad (21)$$

subject to

$$[\mu_z(X)] \leq \lambda \text{ for each objective function } z=1, 2, \dots, O_f$$

$$\begin{aligned}
A X &\leq B & (22) \\
0 \leq \lambda &\leq 1 & (23) \\
X &\geq 0 & (24) \\
X &\geq 0 & (25)
\end{aligned}$$

RESULTS AND DISCUSSION

Individual optimization

Salient points as observed from individual optimization of objective functions are presented in Table 1. In all three objectives, allocation of three enterprises Dairy, Piggery, Poultry are same i.e., 0.4, 0.01, 0.05 ha (fixed in the model), whereas fishery, agro-forestry and crop (rice-wheat) are varying in the range of 1.5 to 4 ha; 1.0 to 1.54 ha; 2 to 4 ha. However, all three individual objectives have satisfied the minimum specified requirements of enterprises (as kept in the model). Capital requirements are varying in the range of ₹434 807 to ₹543 003; labour employment in the range of 756 to 1018 man-days; farm income in the range of ₹532 210 to ₹689 634. These values also form the basis for choosing lower and upper limits for each objective. It is also observed from Table 1 that the three objectives CR, LE, FI are conflicting (except in farm income values while optimizing labour employment and capital requirement which are same i.e., 532 210) in all aspects. This situation necessitates formulating compromise plan(s), and FLP model is found to be suitable to analyze the above situation.

Formulation and application of fuzzy linear programming model to the case study

Formulation of membership functions for the case study based on equation 16 and Table 1 are:

$$\mu_{CR}(X) = \left[ABS\left(\frac{CR_U - CR(X)}{CR_U - CR_L}\right) \right]^\beta \text{ for } CR_L \leq CR(X) \leq CR_U \quad (26)$$

$$\mu_{CR}(X) = 1 \text{ for } CR(X) \leq CR_L$$

labour employment (minimization)

$$\mu_{LE}(X) = 0 \text{ for } LE(X) \geq LE_U$$

$$\mu_{LE}(X) = \left[ABS\left(\frac{LE_U - LE(X)}{LE_U - LE_L}\right) \right]^\beta \text{ for } LE_L \leq LE(X) \leq LE_U \quad (27)$$

$$\mu_{LE}(X) = 1 \text{ for } LE(X) \leq LE_L$$

farm income (maximization)

$$\mu_{FI}(X) = 0 \text{ for } FI(X) \leq FI_L$$

$$\mu_{FI}(X) = \left[ABS\left(\frac{FI(X) - FI_L}{FI_U - FI_L}\right) \right]^\beta \text{ for } FI_L \leq FI(X) \leq FI_U \quad (28)$$

$$\mu_{FI}(X) = 1 \text{ for } FI(X) \geq FI_U$$

Here ABS represents absolute value, are membership functions for capital requirement, labour employment and farm income respectively. These are the upper and lower bounds respectively for objectives capital requirement, labour employment, farm income which are obtained from Table 1. Substituting values in equations 26 to 28 results corresponding membership function values which are given below.

Capital requirements (minimization)

$$\mu_{CR}(X) = 0 \text{ for } CR(X) \geq 543003$$

$$\mu_{CR}(X) = \left[ABS\left(\frac{543003 - CR(X)}{543003 - 434807}\right) \right]^\beta \text{ for } 434807 \leq CR(X) \leq 543003 \quad (29)$$

$$\mu_{CR}(X) = 1 \text{ for } CR(X) \leq 434807$$

Labour employment (minimization)

$$\mu_{LE}(X) = 0 \text{ for } LE(X) \geq 1018$$

$$\mu_{LE}(X) = \left[ABS\left(\frac{1018 - LE(X)}{1018 - 756}\right) \right]^\beta \text{ for } 756 \leq LE(X) \leq 1018 \quad (30)$$

$$\mu_{LE}(X) = 1 \text{ for } LE(X) \leq 756$$

Farm income (maximization)

$$\mu_{FI}(X) = 0 \text{ for } FI(X) \leq 532210$$

$$\mu_{FI}(X) = \left[ABS\left(\frac{FI(X) - 532210}{689634 - 532210}\right) \right]^\beta \text{ for } 532210 \leq FI(X) \leq 689634 \quad (31)$$

$$\mu_{FI}(X) = 1 \text{ for } FI(X) \geq 689634$$

Formulation of mathematical model in FLP in multi-objective framework based on equations 21 to 25 are:

$$\text{Max } \lambda \quad (32)$$

Subject to

$$\left[ABS\left(\frac{543003 - CR(X)}{543003 - 434807}\right) \right]^\beta \geq \lambda \quad (33)$$

$$\left[ABS\left(\frac{1018 - LE(X)}{1018 - 756}\right) \right]^\beta \geq \lambda \quad (34)$$

$$\left[ABS\left(\frac{FI(X) - 532210}{689634 - 532210}\right) \right]^\beta \geq \lambda \quad (35)$$

$$0 \leq \lambda \leq 1 \quad (36)$$

Fixing $b = 1$ results in linear membership function whereas other than 1 results in nonlinear membership function.

Optimization through FLP

Results of FLP are presented in Table 1. It is observed from FLP solution that in all three objectives, allocation of

three enterprises Dairy, Piggery, Poultry are same i.e., 0.4, 0.01, 0.05 ha (fixed in the model), whereas fishery, agro forestry and crop (rice-wheat) are 3.33 ha (in the range of 1.5 to 4 ha obtained from the individual optimal solutions), 1 ha (in the range of 1.0 to 1.54 ha obtained from the individual optimal solutions), 3.34 ha (in the range of 2 to 4 ha obtained from the individual optimal solutions). It is observed from Table 1 (with reference to solution of FLP) that capital requirements, labour requirements, farm income are ₹493 071 (in the range of ₹434 807 to ₹543 003 obtained from the individual optimal solutions); 897 man-days (in the range of 756 to 1018 man-days obtained from the individual optimal solutions); ₹604 860 (in the range of ₹532 210 to ₹689 634 obtained from the individual optimal solutions) with λ value of 0.462 since FLP solution is a compromise solution for three conflicting objectives.

Sensitivity analysis

Sensitivity analysis is performed for different exponential values of β i.e., 0.5 and 2 ($\beta=1$ is also used for comparative analysis) in membership functions to consider nonlinearity. It is observed from Table 2 that λ values are 0.679, 0.462 and 0.213 (decreasing for $\beta=0.5, 1, 2$). It is observed that solution for $\beta=0.5$ and 1 remains same. In case of $\beta=2$, these values are lower as compared to $\beta=1$ scenario and being associated with a lower λ value of 0.213.

Efforts are also made to consider the effect of considering fuzziness only in two of the three objective functions which are as described as follows: In the first scenario of considering capital requirement and farm income as only objectives in fuzzy environment (excluding labour em-

ployment objective), λ values for $\beta=0.5, 1, 2$ are 0.714, 0.509, 0.259 respectively. Solution pattern remains same for $\beta=0.5$ and 1, whereas it is different for $\beta=2$. It is observed from Table 2 (with reference to solution of FLP i.e., $\beta=1$) that capital requirements, labour requirements, farm income are ₹487 886 (in the range of ₹434 807 to ₹543 003 obtained from the individual optimal solutions); 951 man-days (in the range of 756 to 1018 man-days obtained from the individual optimal solutions); ₹612 405 (in the range of ₹532 210 to ₹689 634 obtained from the individual optimal solutions). Land utilization is 8.322 ha whereas food production is 40 tonnes.

In the second scenario of considering farm income and labour employment as objectives in fuzzy environment (excluding capital requirement), λ values for $\beta=0.5, 1, 2$ are 0.747, 0.558, 0.311 respectively. Solution pattern remains same for $\beta=0.5$ and 1 whereas it is different for $\beta=2$. It is observed from Table 2 (with reference to solution of FLP i.e., $\beta=1$) that capital requirements, labour requirements and farm income are ₹510 132, 872 man-days and ₹620 017. Land utilization is 8.34 ha, whereas food production is 28.79 tonnes.

In the third scenario of considering capital requirement and labour employment as objectives in fuzzy environment (excluding farm income), λ values for $\beta=0.5, 1, 2$ are 0.901, 0.811, 0.658 respectively. Solution pattern remains same for $\beta=0.5, 1$ and 2. Capital requirements, labour requirements, farm income for FLP solution are ₹455 240, 806 man-days, ₹532 210, whereas land utilization and food production is 6.832 ha and 26.68 tonnes.

It is inferred from above analysis that the proposed methodology can be extended/replicated to real world planning problems with any number of objective func-

Table 1. Solution obtained by optimization of individual objective functions and fuzzy linear programming

Characteristics	Solution for			
	Min. Capital requirement (CR)	Min. Labour requirement (LE)	Max Farm Income (FI)	FLP ($\beta=1$)
x_1 (dairy) [†]	0.4	0.4	0.4	0.4
x_2 (piggery) [†]	0.01	0.01	0.01	0.01
x_3 (Poultry) [†]	0.05	0.05	0.05	0.05
x_4 (Fishery) [†]	1.50	3.30	4.0	3.33
x_5 (Agro-forestry)	1.0	1.0	1.54	1.0
x_6 (Crop) [†]	4.0	2.0	4.0	3.34
Land utilization (ha)	6.965	6.766	10.0	8.128
Food Requirement (tonnes)	40	20	40	33.36
Capital requirement (CR (₹))	134807*	465 499	543 003	493 071
Labour requirement LE (Man days)	904	756*	1018**	897
Farm Income FI : (₹)	532 210**	532 210	689 634*	604 860
Degree of satisfaction (λ)				0.462

*represent ideal value; **represent anti-ideal value; [†] in ha

Table 2. Fuzzy compromise solutions taking different objectives into consideration

Characteristics	CR, LE and FI			Scenario 1: CR and FI			Scenario 2: LE and FI			Scenario 3: CR and LE		
	$\beta=0.5$	$\beta=1$	$\beta=2$	$\beta=0.5$	$\beta=1$	$\beta=2$	$\beta=0.5$	$\beta=1$	$\beta=2$	$\beta=0.5$	$\beta=1$	$\beta=2$
x_1 (dairy)*	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4
x_2 (piggery) *	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
x_3 (Poultry) *	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05
x_4 (Fishery) *	3.31	3.33	3.31	2.862	2.862	1.51	4.0	4.0	3.306	2.704	2.704	2.70
x_5 (Agro-forestry) *	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
x_6 (Crop) *	3.34	3.34	2.0	4.0	4.0	4.0	2.879	2.879	2.0	2.668	2.668	2.67
Land utilization (ha)	8.128	8.128	6.756	8.322	8.322	6.965	8.34	8.34	6.77	6.832	6.832	6.83
Food Requirement (tonnes)	33.37	33.36	20.0	40.0	40.0	40.0	28.79	28.79	20.0	26.68	26.68	26.68
Capital requirement CR (₹)	493 071	493 071	465 499	487 886	487 886	434 807	510 132	510 132	465 499	455 240	455 240	455 240
Labour requirement LE (Man days)	897	897	756	951	951	904	872	872	756	806	806	806
Farm Income FI : (₹)	604 860	604 860	532 210	612 405	612 405	532 210	620 017	620 017	532 210	532 210	532 210	532 210
λ value	0.679	0.462	0.213	0.714	0.509	0.259	0.747	0.558	0.311	0.901	0.811	0.658

*area in ha

tions, constraints and bounds with simple modifications.

Overall, the present paper discussed the application of Fuzzy Linear Programming (FLP) in integrated farming system scenario in multi-objective framework. The methodology provides a suitable mechanism for integrating economic and social related aspects into the planning problem. FLP methodology is advantageous as compared to single objective planning problems as it can incorporate any number of objectives with ease and without much computational burden. The additional and potential advantage is its ability to consider the uncertainty aspects in the form of membership functions. In the present study extensive sensitivity analysis has also been performed for different values and for different scenarios. The study can be very well extended to the situations of marginal, small and medium farmers with similar or different objective functions (linear/nonlinear). From the study it is further observed that effect of β values are having significant impact on λ values as is increasing or decreasing. However, solution pattern and proposed methodology almost remains same.

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